THREE-DIMENSIONAL NON-REFLECTING BOUNDARY CONDITION FOR LINEARIZED FLOW SOLVERS

Paul Petrie-Repar

RPMTurbo Pty Ltd

TURBO EXPO 2010
THREE-DIMENSIONAL NON-REFLECTING BOUNDARY CONDITION FOR LINEARIZED FLOW SOLVERS

- Linearised 3D Viscous Flow Solver
- Exact 3D Non-Reflecting Boundary Condition
- Aero-acoustic Test Case: Category 4 Benchmark Problem from 3rd CAA Workshop
- Flutter Test Case: 3D Standard Configuration 10
Philosophy

- Steady flow condition known
- Flutter: known grid motion: $x = \bar{x} + \tilde{x} e^{j\omega t}$
- Acoustics: known incoming disturbance
- Discretised unsteady flow model: $\frac{dU}{dt} = R(U, x, \dot{x})$
- Unknown flow perturbation: $U = \overline{U} + \tilde{U} e^{j\omega t}$
- Linearisation: $R \approx \overline{R} + \frac{\partial R}{\partial U} \Delta U + \frac{\partial R}{\partial x} \Delta x + \frac{\partial R}{\partial \dot{x}} \Delta \dot{x}$
  $$[j \omega - \frac{\partial R}{\partial U}] \tilde{U} \approx R(\overline{U}, \bar{x} + \tilde{x}, 0) + j \omega R(\overline{U}, \bar{x}, \tilde{x})$$
- 100 to 1000 times faster than time domain methods
- Single passage for turbomachinery
- Can apply exact non-reflecting boundary conditions
RPMTurbo Linearised Flow Solver

- 3D viscous flow with Spalart and Allmaras turbulence model
- Efficient parallel solver for linear systems
- 3D Euler 140,000 cells in 2 minutes (10 procs.)
- 3D Viscous 500,000 cells in 20 minutes (30 procs.)
- Validated - Standard Configuration 10 and 11
- Non-reflecting boundary condition
Non-Reflecting Boundary Condition

Philosophy

- Allow outgoing waves to exit domain without reflection
- Reflected waves can pollute solution
- Decompose unsteady flow into waves (modes)
- 2D and 3D flow: must consider entire boundary
- Determine direction of each wave
- Prescribe incoming waves
- Extrapolate outgoing waves
Current Methods

- Commercial Software: use steady boundary conditions
- Assume 1D waves: apply locally
- Giles: 2D analytical modes for uniform flow
- Strip Method: apply 2D method at radial slices
- Hall/Montgomery: numerically determine 3D modes
Numerically determine aerodynamic modes at far-field

- Create 2D mesh for far-field
- Semi-discretized flow equations
  \[ \frac{\partial U_f}{\partial t} = A_f \frac{\partial U_f}{\partial x} + D_f U_f \]
- Assuming wave-like solution
  \[ U_f = U_m(y, z) \exp \{i(kx + \omega t)\} \]
- Solve eigen problem to determine modes
  \[ A_f^{-1} [\omega I + iD_f] U_m = k U_m \]
- Steady flow at far-field can be non-uniform and swirling
### Aeroacoustic Test Case

Category 4 Benchmark Problem from 3rd CAA Workshop

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of vanes</td>
<td>24</td>
</tr>
<tr>
<td>Number of blades</td>
<td>16</td>
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<tr>
<td>Axial Mach number</td>
<td>0.5</td>
</tr>
<tr>
<td>Hub to tip ratio</td>
<td>0.5</td>
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<tr>
<td>Gap to chord ratio at tip</td>
<td>1.0</td>
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<tr>
<td>Mach tip (incoming wake)</td>
<td>0.783</td>
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<tr>
<td>Incoming amplitude</td>
<td>0.1 $U_\infty$</td>
</tr>
</tbody>
</table>
Aeroacoustic Test Case

Example Far-field Acoustic Modes

(-8,0)  (-8,1)  (-8,2)  (-8,3)  (16,0)  (16,1)
Aeroacoustic Test Case

Category 4 Benchmark Problem from 3rd CAA Workshop: Case 4

Wave plot

Pressure (dB)

Outgoing

Incoming

(16,0)

(16,1)

(16,2)

(16,3)

(16,4)

(-8,0)

(-8,1)

(-8,2)

(-8,3)

(-8,4)

x

Wave plot

Non-Reflecting Boundary Condition

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## Aeroacoustic Test Case

### Category 4 Benchmark Problem from 3rd CAA Workshop: Case 4

<table>
<thead>
<tr>
<th>$m$</th>
<th>$\mu$</th>
<th>Namba</th>
<th>Schulten</th>
<th>RPMTurbo</th>
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<tbody>
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<td>7.064E-04</td>
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<td>1.174E-02</td>
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<tr>
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<td>5.283E-06</td>
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<td>8.079E-06</td>
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</table>

Case 4: Amplitudes of Outgoing Acoustic Modes at Inlet
Category 4 Benchmark Problem from 3rd CAA Workshop: Case 4

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Namba</th>
<th>Schulten</th>
<th>RPMTurbo</th>
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<td>1.045E-04</td>
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<td>1.025E-05</td>
<td>1.230E-05</td>
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<tr>
<td>16</td>
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<td>2.237E-06</td>
<td>2.375E-06</td>
<td>2.393E-06</td>
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<tr>
<td>16</td>
<td>3</td>
<td>4.373E-07</td>
<td>5.301E-07</td>
<td>4.354E-07</td>
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<tr>
<td>-8</td>
<td>0</td>
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<td>1.497E-02</td>
<td>1.243E-02</td>
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<tr>
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<td>1.785E-02</td>
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<td>2.804E-06</td>
<td>4.294E-06</td>
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</table>

Case 4: Amplitudes of Outgoing Acoustic Modes at Exit
Flutter Test Case

3D Standard Configuration 10

Geometry and Flow Conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Blades</td>
<td>24</td>
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<tr>
<td>Blade Shape</td>
<td>untwisted</td>
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<tr>
<td>Chord Length</td>
<td>100 mm</td>
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<tr>
<td>Hub Radius</td>
<td>339.5 mm</td>
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<tr>
<td>Shroud Radius</td>
<td>424.4 mm</td>
</tr>
<tr>
<td>Stagger Angle</td>
<td>45.0°</td>
</tr>
<tr>
<td>Inlet Mach Number</td>
<td>0.7</td>
</tr>
<tr>
<td>Inlet Flow Angle</td>
<td>55.0°</td>
</tr>
<tr>
<td>Reynolds Number</td>
<td>$1.25 \times 10^6$</td>
</tr>
</tbody>
</table>
Unsteady Linearised Inviscid Flow Solution with 3D NRBC

Damping Plot for Torsion Mode ($\omega^* = 0.5$)
3D Standard Configuration 10

Short Flow Domain: Far-field @ 0.2 chord
Unsteady Linearised Inviscid Flow Solution with 3D NRBC

Damping Plot for Torsion Mode ($\omega^* = 0.5$)
Unsteady Linearised Inviscid Flow Solution with 1D NRBC

Damping Plot for Torsion Mode \( (\omega^* = 0.5) \)
Unsteady Linearised Inviscid Flow Solution with 1D NRBC

Wave Plot for Torsion Mode ($\sigma = 15^\circ$, $\omega^* = 0.5$)
Unsteady Linearised Inviscid Flow Solution with 3D NRBC

Wave Plot for Torsion Mode ($\sigma = 15^\circ$, $\omega^* = 0.5$)
Conclusions

- Exact 3D non-reflecting boundary condition has been developed
- Aeroacoustic test case: Category 4 Benchmark Problem. Solution agrees well with other solutions.
- Flutter test case: 3D Standard Configuration 10. Solution independent of far-field location.